2.3 Logic Puzzles

MATERIALS • graph paper • pencils

QUESTION How can reasoning be used to solve a logic puzzle?

EXPLORE Solve a logic puzzle

Using the clues below, you can determine an important mathematical contribution and interesting fact about each of five mathematicians.

Copy the chart onto your graph paper. Use the chart to keep track of the information given in Clues 1–7. Place an X in a box to indicate a definite "no." Place an O in a box to indicate a definite "yes."

Clue 1 Pythagoras had his contribution named after him. He was known to avoid eating beans.

Clue 2 Albert Einstein considered Emmy Noether to be one of the greatest mathematicians and used her work to show the theory of relativity.

Clue 3 Anaxagoras was the first to theorize that the moon's light is actually the sun's light being reflected.

Clue 4 Julio Rey Pastor wrote a book at age 17.

Clue 5 The mathematician who is fluent in Latin contributed to the study of differential calculus.

Clue 6 The mathematician who did work with *n*-dimensional geometry was not the piano player.

Clue 7 The person who first used perspective drawing to make scenery for plays was not Maria Agnesi or Julio Rey Pastor.

	n-dimensional geometry	Differential calculus	Math for theory of relativity	Perspective drawing	Pythagorean Theorem	Did not eat beans	Studied moonlight	Wrote a math book at 17	Fluent in Latin	Played piano
Maria Agnesi					X					
Anaxagoras					X					
Emmy Noether					X					
Julio Rey Pastor					X					
Pythagoras	X	X	X	X	O					
Did not eat beans										
Studied moonlight										
Wrote a math book at 17										
Fluent in Latin										
Played piano										

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Write Clue 4 as a conditional statement in if-then form. Then write the contrapositive of the statement. *Explain* why the contrapositive of this statement is a helpful clue.

2. *Explain* how you can use Clue 6 to figure out who played the piano.

3. *Explain* how you can use Clue 7 to figure out who worked with perspective drawing.

2.3 Apply Deductive Reasoning

Before You used inductive reasoning to form a conjecture.

Now You will use deductive reasoning to form a logical argument.

Why So you can reach logical conclusions about locations, as in Ex. 18.

Key Vocabulary
- **deductive reasoning**

Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to form a logical argument. This is different from *inductive reasoning*, which uses specific examples and patterns to form a conjecture.

READ VOCABULARY
The Law of Detachment is also called a *direct argument*. The Law of Syllogism is sometimes called the *chain rule*.

KEY CONCEPT *For Your Notebook*

Laws of Logic

Law of Detachment

If the hypothesis of a true conditional statement is true, then the conclusion is also true.

Law of Syllogism

If **hypothesis** ***p***, then **conclusion** ***q***.
If **hypothesis** ***q***, then **conclusion** ***r***. — If these statements are true,

If **hypothesis** ***p***, then **conclusion** ***r***. — then this statement is true.

EXAMPLE 1 Use the Law of Detachment

Use the Law of Detachment to make a valid conclusion in the true situation.

a. If two segments have the same length, then they are congruent. You know that $BC = XY$.

b. Mary goes to the movies every Friday and Saturday night. Today is Friday.

Solution

a. Because $BC = XY$ satisfies the hypothesis of a true conditional statement, the conclusion is also true. So, $\overline{BC} \cong \overline{XY}$.

b. First, identify the hypothesis and the conclusion of the first statement. The hypothesis is "If it is Friday or Saturday night," and the conclusion is "then Mary goes to the movies."

"Today is Friday" satisfies the hypothesis of the conditional statement, so you can conclude that Mary will go to the movies tonight.

EXAMPLE 2 Use the Law of Syllogism

If possible, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements.

a. If Rick takes chemistry this year, then Jesse will be Rick's lab partner.

If Jesse is Rick's lab partner, then Rick will get an A in chemistry.

b. If $x^2 > 25$, then $x^2 > 20$.

If $x > 5$, then $x^2 > 25$.

c. If a polygon is regular, then all angles in the interior of the polygon are congruent.

If a polygon is regular, then all of its sides are congruent.

Solution

a. The conclusion of the first statement is the hypothesis of the second statement, so you can write the following new statement.

If Rick takes chemistry this year, then Rick will get an A in chemistry.

AVOID ERRORS
The order in which the statements are given does not affect whether you can use the Law of Syllogism.

b. Notice that the conclusion of the second statement is the hypothesis of the first statement, so you can write the following new statement.

If $x > 5$, then $x^2 > 20$.

c. Neither statement's conclusion is the same as the other statement's hypothesis. You cannot use the Law of Syllogism to write a new conditional statement.

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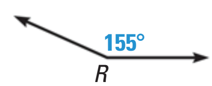
✓ GUIDED PRACTICE for Examples 1 and 2

1. If $90° < m\angle R < 180°$, then $\angle R$ is obtuse. The measure of $\angle R$ is 155°. Using the Law of Detachment, what statement can you make?



2. If Jenelle gets a job, then she can afford a car. If Jenelle can afford a car, then she will drive to school. Using the Law of Syllogism, what statement can you make?

State the law of logic that is illustrated.

3. If you get an A or better on your math test, then you can go to the movies. If you go to the movies, then you can watch your favorite actor.

If you get an A or better on your math test, then you can watch your favorite actor.

4. If $x > 12$, then $x + 9 > 20$. The value of x is 14.

Therefore, $x + 9 > 20$.

ANALYZING REASONING In Geometry, you will frequently use inductive reasoning to make conjectures. You will also be using deductive reasoning to show that conjectures are true or false. You will need to know which type of reasoning is being used.

EXAMPLE 3 Use inductive and deductive reasoning

xy ALGEBRA What conclusion can you make about the product of an even integer and any other integer?

Solution

STEP 1 **Look** for a pattern in several examples. Use inductive reasoning to make a conjecture.

$(-2)(2) = -4$, $(-1)(2) = -2$, $2(2) = 4$, $3(2) = 6$,

$(-2)(-4) = 8$, $(-1)(-4) = 4$, $2(-4) = -8$, $3(-4) = -12$

Conjecture Even integer • Any integer = Even integer

STEP 2 **Let** n and m each be any integer. Use deductive reasoning to show the conjecture is true.

$2n$ is an even integer because any integer multiplied by 2 is even.

$2nm$ represents the product of an even integer and any integer m.

$2nm$ is the product of 2 and an integer nm. So, $2nm$ is an even integer.

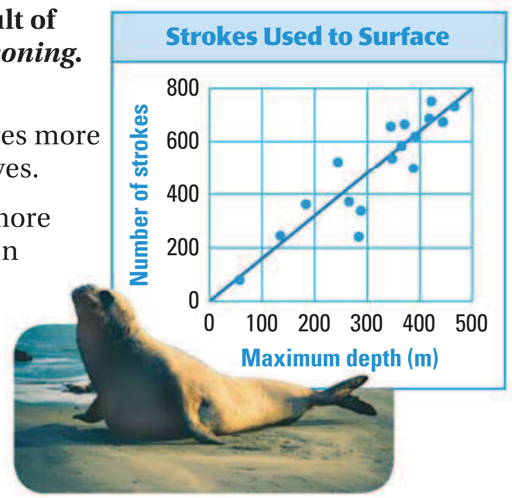
▸ The product of an even integer and any integer is an even integer.

EXAMPLE 4 Reasoning from a graph

Tell whether the statement is the result of *inductive reasoning* or *deductive reasoning*. Explain your choice.

a. The northern elephant seal requires more strokes to surface the deeper it dives.

b. The northern elephant seal uses more strokes to surface from 60 feet than from 250 feet.



Solution

a. Inductive reasoning, because it is based on a pattern in the data

b. Deductive reasoning, because you are comparing values that are given on the graph

✓ GUIDED PRACTICE for Examples 3 and 4

5. Use inductive reasoning to make a conjecture about the sum of a number and itself. Then use deductive reasoning to show the conjecture is true.

6. Use inductive reasoning to write another statement about the graph in Example 4. Then use deductive reasoning to write another statement.

2.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 17, and 21

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 3, 12, 20, and 23

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: If the hypothesis of a true if-then statement is true, then the conclusion is also true by the Law of _?_.

★ **WRITING** **Use deductive reasoning to make a statement about the picture.**

2. 

3. 

EXAMPLE 1 on p. 87 for Exs. 4–6

LAW OF DETACHMENT **Make a valid conclusion in the situation.**

4. If the measure of an angle is 90°, then it is a right angle. The measure of $\angle A$ is 90°.

5. If $x > 12$, then $-x < -12$. The value of x is 15.

6. If a book is a biography, then it is nonfiction. You are reading a biography.

EXAMPLE 2 on p. 88 for Exs. 7–10

LAW OF SYLLOGISM **In Exercises 7–10, write the statement that follows from the pair of statements that are given.**

7. If a rectangle has four equal side lengths, then it is a square. If a polygon is a square, then it is a regular polygon.

8. If $y > 0$, then $2y > 0$. If $2y > 0$, then $2y - 5 \neq -5$.

9. If you play the clarinet, then you play a woodwind instrument. If you play a woodwind instrument, then you are a musician.

10. If $a = 3$, then $5a = 15$. If $\frac{1}{2}a = 1\frac{1}{2}$, then $a = 3$.

EXAMPLE 3 on p. 89 for Ex. 11

11. **REASONING** What can you say about the sum of an even integer and an even integer? Use inductive reasoning to form a conjecture. Then use deductive reasoning to show that the conjecture is true.

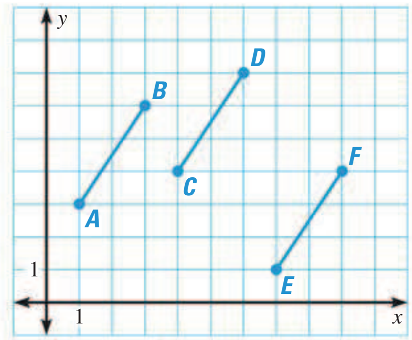
12. ★ **MULTIPLE CHOICE** If two angles are vertical angles, then they have the same measure. You know that $\angle A$ and $\angle B$ are vertical angles. Using the Law of Detachment, which conclusion could you make?

    (A) $m\angle A > m\angle B$ (B) $m\angle A = m\angle B$

    (C) $m\angle A + m\angle B = 90°$ (D) $m\angle A + m\angle B = 180°$

13. **ERROR ANALYSIS** *Describe* and correct the error in the argument: "If two angles are a linear pair, then they are supplementary. Angles C and D are supplementary, so the angles are a linear pair."

14. xy **ALGEBRA** Use the segments in the coordinate plane.



a. Use the distance formula to show that the segments are congruent.

b. Make a conjecture about some segments in the coordinate plane that are congruent to the given segments. Test your conjecture, and *explain* your reasoning.

c. Let one endpoint of a segment be (x, y). Use algebra to show that segments drawn using your conjecture will always be congruent.

d. A student states that the segments described below will each be congruent to the ones shown above. Determine whether the student is correct. *Explain* your reasoning.

$\overline{MN}$, with endpoints $M(3, 5)$ and $N(5, 2)$

$\overline{PQ}$, with endpoints $P(1, -1)$ and $Q(4, -3)$

$\overline{RS}$, with endpoints $R(-2, 2)$ and $S(1, 4)$

15. **CHALLENGE** Make a conjecture about whether the Law of Syllogism works when used with the contrapositives of a pair of statements. Use this pair of statements to *justify* your conjecture.

If a creature is a wombat, then it is a marsupial.

If a creature is a marsupial, then it has a pouch.

PROBLEM SOLVING

EXAMPLES 1 and 2 on pp. 87–88 for Exs. 16–17

USING THE LAWS OF LOGIC **In Exercises 16 and 17, what conclusions can you make using the true statement?**

16. **CAR COSTS** If you save \$2000, then you can buy a car. You have saved \$1200.

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17. **PROFIT** The bakery makes a profit if its revenue is greater than its costs. You will get a raise if the bakery makes a profit.

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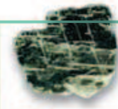
USING DEDUCTIVE REASONING **Select the word(s) that make(s) the conclusion true.**

18. Mesa Verde National Park is in Colorado. Simone vacationed in Colorado. So, Simone (*must have*, *may have*, or *never*) visited Mesa Verde National Park.

19. The cliff dwellings in Mesa Verde National Park are accessible to visitors only when accompanied by a park ranger. Billy is at a cliff dwelling in Mesa Verde National Park. So, Billy *(is, may be, is not)* with a park ranger.

EXAMPLE 4
on p. 89
for Ex. 20

20. ★ **EXTENDED RESPONSE** Geologists use the Mohs scale to determine a mineral's hardness. Using the scale, a mineral with a higher rating will leave a scratch on a mineral with a lower rating. Geologists use scratch tests to help identify an unknown mineral.

Mineral	 Talc	 Gypsum	 Calcite	 Fluorite
Mohs rating	1	2	3	4

a. Use the table to write three if-then statements such as "If talc is scratched against gypsum, then a scratch mark is left on the talc."

b. You must identify four minerals labeled *A*, *B*, *C*, and *D*. You know that the minerals are the ones shown in the table. The results of your scratch tests are shown below. What can you conclude? *Explain* your reasoning.

Mineral *A* is scratched by Mineral *B*.

Mineral *C* is scratched by all three of the other minerals.

c. What additional test(s) can you use to identify *all* the minerals in part (b)?

REASONING **In Exercises 21 and 22, decide whether *inductive* or *deductive* reasoning is used to reach the conclusion. *Explain* your reasoning.**

21. The rule at your school is that you must attend all of your classes in order to participate in sports after school. You played in a soccer game after school on Monday. Therefore, you went to all of your classes on Monday.

22. For the past 5 years, your neighbor goes on vacation every July 4th and asks you to feed her hamster. You conclude that you will be asked to feed her hamster on the next July 4th.

23. ★ **SHORT RESPONSE** Let an even integer be $2n$ and an odd integer be $2n + 1$. *Explain* why the sum of an even integer and an odd integer is an odd integer.

24. **LITERATURE** George Herbert wrote a poem, *Jacula Prudentum*, that includes the statements shown. Use the Law of Syllogism to write a new conditional statement. *Explain* your reasoning.

For want of a nail the shoe is lost,
for want of a shoe the horse is lost,
for want of a horse the rider is lost.

REASONING **In Exercises 25–28, use the true statements below to determine whether you know the conclusion is *true* or *false*. *Explain* your reasoning.**

If Arlo goes to the baseball game, then he will buy a hot dog.

If the baseball game is not sold out, then Arlo and Mia will go to the game.

If Mia goes to the baseball game, then she will buy popcorn.

The baseball game is not sold out.

25. Arlo bought a hot dog.

26. Arlo and Mia went to the game.

27. Mia bought a hot dog.

28. Arlo had some of Mia's popcorn.

29. **CHALLENGE** Use these statements to answer parts (a)–(c).

Adam says Bob lies.

Bob says Charlie lies.

Charlie says Adam and Bob both lie.

a. If Adam is telling the truth, then Bob is lying. What can you conclude about Charlie's statement?

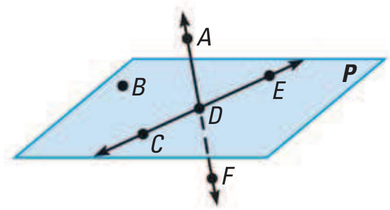
b. Assume Adam is telling the truth. *Explain* how this leads to a contradiction.

c. Who is telling the truth? Who is lying? How do you know?

MIXED REVIEW

PREVIEW
Prepare for Lesson 2.4 in Exs. 30–33.

In Exercises 30–33, use the diagram. *(p. 2)*



30. Name two lines.

31. Name four rays.

32. Name three collinear points.

33. Name four coplanar points.

Plot the given points in a coordinate plane. Then determine whether $\overline{AB}$ and $\overline{CD}$ are congruent. *(p. 9)*

34. $A(1, 4), B(5, 4), C(3, -4), D(3, 0)$

35. $A(-1, 0), B(-1, -5), C(1, 2), D(-5, 2)$

Rewrite the conditional statement in if-then form. *(p. 79)*

36. When $x = -2$, $x^2 = 4$.

37. The measure of an acute angle is less than 90°.

38. Only people who are members can access the website.

QUIZ *for Lessons 2.1–2.3*

Show the conjecture is false by finding a counterexample. *(p. 72)*

1. If the product of two numbers is positive, then the two numbers must be negative.

2. The sum of two numbers is always greater than the larger number.

In Exercises 3 and 4, write the if-then form and the contrapositive of the statement. *(p. 79)*

3. Points that lie on the same line are called collinear points.

4. $2x - 8 = 2$, because $x = 5$.

5. Make a valid conclusion about the following statements:

If it is above 90°F outside, then I will wear shorts. It is 98°F. *(p. 87)*

6. *Explain* why a number that is divisible by a multiple of 3 is also divisible by 3. *(p. 87)*

Extension

Use after Lesson 2.3

Symbolic Notation and Truth Tables

GOAL Use symbolic notation to represent logical statements.

Key Vocabulary
- **truth value**
- **truth table**

Conditional statements can be written using *symbolic notation*, where letters are used to represent statements. An arrow ($\rightarrow$), read "implies," connects the hypothesis and conclusion. To write the negation of a statement p you write the symbol for negation ($\sim$) before the letter. So, "not p" is written $\sim p$.

KEY CONCEPT — *For Your Notebook*

Symbolic Notation

Let p be "the angle is a right angle" and let q be "the measure of the angle is 90°."

Conditional	If p, then q.	$p \rightarrow q$

Example: If an angle is a right angle, then its measure is 90°.

Converse	If q, then p.	$q \rightarrow p$

Example: If the measure of an angle is 90°, then the angle is a right angle.

Inverse	If not p, then not q.	$\sim p \rightarrow \sim q$

Example: If an angle is not a right angle, then its measure is not 90°.

Contrapositive	If not q, then not p.	$\sim q \rightarrow \sim p$

If the measure of an angle is not 90°, then the angle is not a right angle.

Biconditional	p if and only if q	$p \leftrightarrow q$

Example: An angle is a right angle if and only if its measure is 90°.

EXAMPLE 1 Use symbolic notation

Let p be "the car is running" and let q be "the key is in the ignition."

a. Write the conditional statement $p \rightarrow q$ in words.

b. Write the converse $q \rightarrow p$ in words.

c. Write the inverse $\sim p \rightarrow \sim q$ in words.

d. Write the contrapositive $\sim q \rightarrow \sim p$ in words.

Solution

a. Conditional: If the car is running, then the key is in the ignition.

b. Converse: If the key is in the ignition, then the car is running.

c. Inverse: If the car is not running, then the key is not in the ignition.

d. Contrapositive: If the key is not in the ignition, then the car is not running.

TRUTH TABLES The **truth value** of a statement is either true (T) or false (F). You can determine the conditions under which a conditional statement is true by using a **truth table**. The truth table at the right shows the truth values for hypothesis p and conclusion q. The conditional $p \rightarrow q$ is only false when a true hypothesis produces a false conclusion.

Conditional		
p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

EXAMPLE 2 Make a truth table

Use the truth table above to make truth tables for the converse, inverse, and contrapositive of a conditional statement $p \rightarrow q$.

Solution

READ TRUTH TABLES
A conditional statement and its contrapositive are *equivalent statements* because they have the same truth table. The same is true of the converse and the inverse.

Converse		
p	q	$q \rightarrow p$
T	T	T
T	F	T
F	T	F
F	F	T

Inverse				
p	q	$\sim p$	$\sim q$	$\sim p \rightarrow \sim q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Contrapositive				
p	q	$\sim q$	$\sim p$	$\sim q \rightarrow \sim p$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

PRACTICE

EXAMPLE 1
on p. 94
for Exs. 1–6

1. **WRITING** *Describe* how to use symbolic notation to represent the contrapositive of a conditional statement.

WRITING STATEMENTS **Use p and q to write the symbolic statement in words.**

p: Polygon *ABCDE* is equiangular and equilateral.

q: Polygon *ABCDE* is a regular polygon.

2. $p \rightarrow q$
3. $\sim p$
4. $\sim q \rightarrow \sim p$
5. $p \leftrightarrow q$

6. **LAW OF SYLLOGISM** Use the statements p, q, and r below to write a series of conditionals that would satisfy the Law of Syllogism. How could you write your reasoning using symbolic notation?

p: $x + 5 = 12$ $\quad$ q: $x = 7$ $\quad$ r: $3x = 21$

EXAMPLE 2
on p. 95
for Exs. 7–8

7. **WRITING** Is the truth value of a statement always true (T)? *Explain*.

8. **TRUTH TABLE** Use the statement "If an animal is a poodle, then it is a dog."
   a. Identify the hypothesis p and the conclusion q in the conditional.
   b. Make a truth table for the converse. *Explain* what each row in the table means in terms of the original statement.