

1.3 Use Midpoint and Distance Formulas



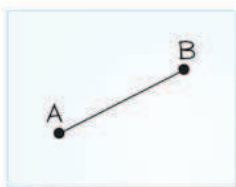
Before You found lengths of segments.
Now You will find lengths of segments in the coordinate plane.
Why? So you can find an unknown length, as in Example 1.

Key Vocabulary

- midpoint
- segment bisector

ACTIVITY FOLD A SEGMENT BISECTOR

STEP 1



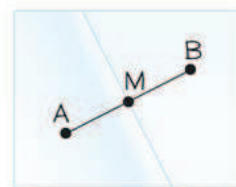
Draw \overline{AB} on a piece of paper.

STEP 2



Fold the paper so that B is on top of A .

STEP 3



Label point M . Compare AM , MB , and AB .

MIDPOINTS AND BISECTORS The **midpoint** of a segment is the point that divides the segment into two congruent segments. A **segment bisector** is a point, ray, line, line segment, or plane that intersects the segment at its midpoint. A midpoint or a segment bisector *bisects* a segment.



M is the midpoint of \overline{AB} .
 So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.



\overleftrightarrow{CD} is a segment bisector of \overline{AB} .
 So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.

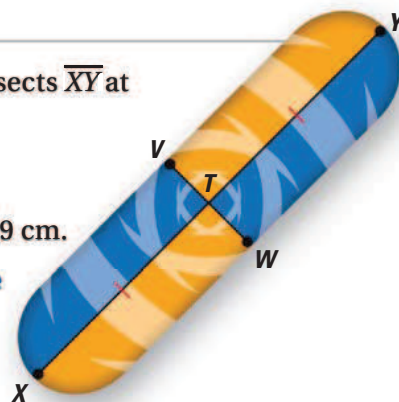
EXAMPLE 1 Find segment lengths

SKATEBOARD In the skateboard design, \overline{VW} bisects \overline{XY} at point T , and $XT = 39.9$ cm. Find XY .

Solution

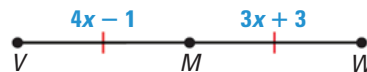
Point T is the midpoint of \overline{XY} . So, $XT = TY = 39.9$ cm.

$$\begin{aligned} XY &= XT + TY && \text{Segment Addition Postulate} \\ &= 39.9 + 39.9 && \text{Substitute.} \\ &= 79.8 \text{ cm} && \text{Add.} \end{aligned}$$



EXAMPLE 2 Use algebra with segment lengths

xy ALGEBRA Point M is the midpoint of \overline{VW} . Find the length of \overline{VM} .



Solution

STEP 1 Write and solve an equation. Use the fact that $VM = MW$.

$$VM = MW \quad \text{Write equation.}$$

$$4x - 1 = 3x + 3 \quad \text{Substitute.}$$

$$x - 1 = 3 \quad \text{Subtract } 3x \text{ from each side.}$$

$$x = 4 \quad \text{Add 1 to each side.}$$

STEP 2 Evaluate the expression for VM when $x = 4$.

$$VM = 4x - 1 = 4(4) - 1 = 15$$

► So, the length of \overline{VM} is 15.

CHECK Because $VM = MW$, the length of \overline{MW} should be 15. If you evaluate the expression for MW , you should find that $MW = 15$.

$$MW = 3x + 3 = 3(4) + 3 = 15 \quad \checkmark$$

REVIEW ALGEBRA

For help with solving equations, see p. 875.

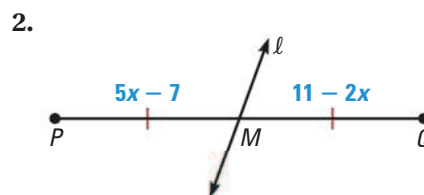
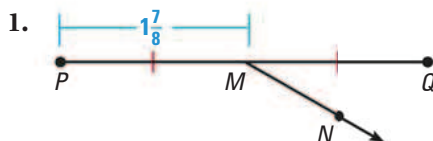


GUIDED PRACTICE for Examples 1 and 2

READ DIRECTIONS

Always read direction lines carefully. Notice that this direction line has two parts.

In Exercises 1 and 2, identify the segment bisector of \overline{PQ} . Then find PQ .



COORDINATE PLANE You can use the coordinates of the endpoints of a segment to find the coordinates of the midpoint.

KEY CONCEPT

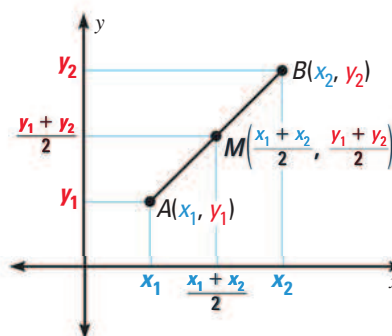
For Your Notebook

The Midpoint Formula

The coordinates of the midpoint of a segment are the averages of the x -coordinates and of the y -coordinates of the endpoints.

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint M of \overline{AB} has coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$



EXAMPLE 3 Use the Midpoint Formula

- a. **FIND MIDPOINT** The endpoints of \overline{RS} are $R(1, -3)$ and $S(4, 2)$. Find the coordinates of the midpoint M .
- b. **FIND ENDPOINT** The midpoint of \overline{JK} is $M(2, 1)$. One endpoint is $J(1, 4)$. Find the coordinates of endpoint K .

Solution

- a. **FIND MIDPOINT** Use the Midpoint Formula.

$$M\left(\frac{1+4}{2}, \frac{-3+2}{2}\right) = M\left(\frac{5}{2}, -\frac{1}{2}\right)$$

- The coordinates of the midpoint M are $\left(\frac{5}{2}, -\frac{1}{2}\right)$.

- b. **FIND ENDPOINT** Let (x, y) be the coordinates of endpoint K . Use the Midpoint Formula.

STEP 1 Find x .

STEP 2 Find y .

$$\frac{1+x}{2} = 2$$

$$\frac{4+y}{2} = 1$$

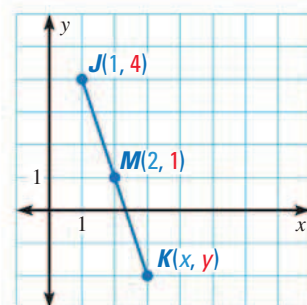
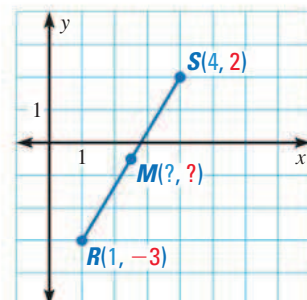
$$1+x = 4$$

$$4+y = 2$$

$$x = 3$$

$$y = -2$$

- The coordinates of endpoint K are $(3, -2)$.



CLEAR FRACTIONS

Multiply each side of the equation by the denominator to clear the fraction.



GUIDED PRACTICE for Example 3

3. The endpoints of \overline{AB} are $A(1, 2)$ and $B(7, 8)$. Find the coordinates of the midpoint M .
4. The midpoint of \overline{VW} is $M(-1, -2)$. One endpoint is $W(4, 4)$. Find the coordinates of endpoint V .

DISTANCE FORMULA The Distance Formula is a formula for computing the distance between two points in a coordinate plane.

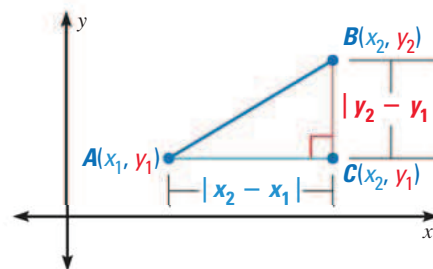
KEY CONCEPT

For Your Notebook

The Distance Formula

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the distance between A and B is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



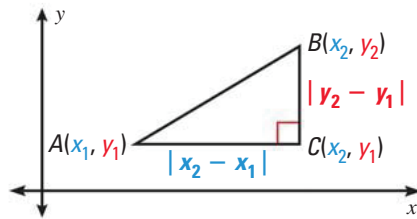
READ DIAGRAMS

The red mark at one corner of the triangle shown indicates a right triangle.

The Distance Formula is based on the *Pythagorean Theorem*, which you will see again when you work with right triangles in Chapter 7.

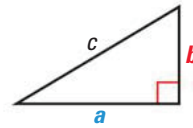
Distance Formula

$$(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$



Pythagorean Theorem

$$c^2 = a^2 + b^2$$



EXAMPLE 4 Standardized Test Practice

ELIMINATE CHOICES

Drawing a diagram can help you eliminate choices. You can see that choice A is not large enough to be RS .

What is the approximate length of \overline{RS} with endpoints $R(2, 3)$ and $S(4, -1)$?

- (A) 1.4 units (B) 4.0 units (C) 4.5 units (D) 6 units

Solution

Use the Distance Formula. You may find it helpful to draw a diagram.

$$\begin{aligned} RS &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[(4 - 2)]^2 + [(-1) - 3]^2} \\ &= \sqrt{(2)^2 + (-4)^2} \\ &= \sqrt{4 + 16} \\ &= \sqrt{20} \\ &\approx 4.47 \end{aligned}$$

Distance Formula

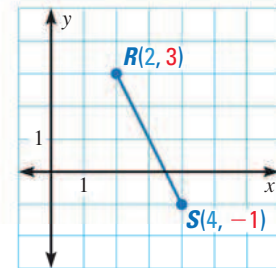
Substitute.

Subtract.

Evaluate powers.

Add.

Use a calculator to approximate the square root.



READ SYMBOLS

The symbol \approx means "is approximately equal to."

▶ The correct answer is C. (A) (B) (C) (D)



GUIDED PRACTICE for Example 4

- In Example 4, does it matter which ordered pair you choose to substitute for (x_1, y_1) and which ordered pair you choose to substitute for (x_2, y_2) ? *Explain.*
- What is the approximate length of \overline{AB} , with endpoints $A(-3, 2)$ and $B(1, -4)$?
(A) 6.1 units (B) 7.2 units (C) 8.5 units (D) 10.0 units


1.3 EXERCISES

HOMEWORK KEY

 = **WORKED-OUT SOLUTIONS**
on p. WS1 for Exs. 15, 35, and 49

 = **STANDARDIZED TEST PRACTICE**
Exs. 2, 23, 34, 41, 42, and 53

SKILL PRACTICE

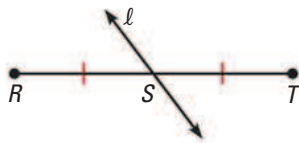
1. **VOCABULARY** Copy and complete: To find the length of \overline{AB} , with endpoints $A(-7, 5)$ and $B(4, -6)$, you can use the .
2.  **WRITING** Explain what it means to bisect a segment. Why is it impossible to bisect a line?

EXAMPLE 1

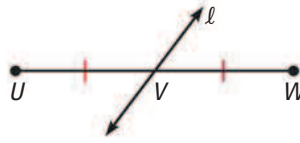
on p. 15
for Exs. 3–10

FINDING LENGTHS Line l bisects the segment. Find the indicated length.

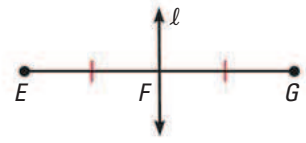
3. Find RT if $RS = 5\frac{1}{8}$ in.



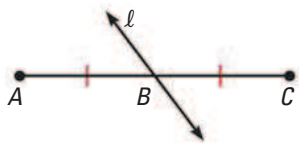
4. Find UW if $VW = \frac{5}{8}$ in.



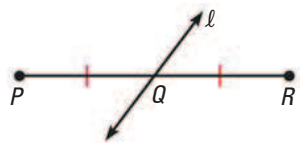
5. Find EG if $EF = 13$ cm.



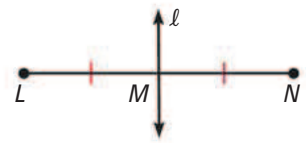
6. Find BC if $AC = 19$ cm.



7. Find QR if $PR = 9\frac{1}{2}$ in.



8. Find LM if $LN = 137$ mm.



9. **SEGMENT BISECTOR** Line RS bisects \overline{PQ} at point R . Find RQ if $PQ = 4\frac{3}{4}$ inches.

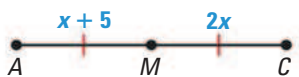
10. **SEGMENT BISECTOR** Point T bisects \overline{UV} . Find UV if $UT = 2\frac{7}{8}$ inches.

EXAMPLE 2

on p. 16
for Exs. 11–16

 **ALGEBRA** In each diagram, M is the midpoint of the segment. Find the indicated length.

11. Find AM .



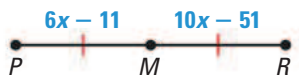
12. Find EM .



13. Find JM .



14. Find PR .



15. Find SU .



16. Find XZ .



EXAMPLE 3

on p. 17
for Exs. 17–30

FINDING MIDPOINTS Find the coordinates of the midpoint of the segment with the given endpoints.

17. $C(3, 5)$ and $D(7, 5)$


18. $E(0, 4)$ and $F(4, 3)$

19. $G(-4, 4)$ and $H(6, 4)$

20. $J(-7, -5)$ and $K(-3, 7)$

21. $P(-8, -7)$ and $Q(11, 5)$

22. $S(-3, 3)$ and $T(-8, 6)$

23.  **WRITING** Develop a formula for finding the midpoint of a segment with endpoints $A(0, 0)$ and $B(m, n)$. Explain your thinking.

24. **ERROR ANALYSIS** Describe the error made in finding the coordinates of the midpoint of a segment with endpoints $S(8, 3)$ and $T(2, -1)$.

$$\left(\frac{8-2}{2}, \frac{3-(-1)}{2}\right) = (3, 2)$$



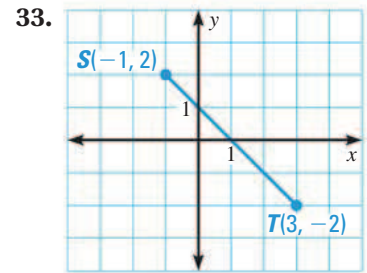
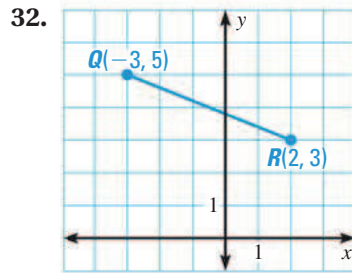
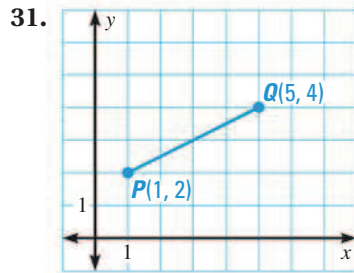
FINDING ENDPOINTS Use the given endpoint R and midpoint M of \overline{RS} to find the coordinates of the other endpoint S .

25. $R(3, 0), M(0, 5)$ 26. $R(5, 1), M(1, 4)$ 27. $R(6, -2), M(5, 3)$
 28. $R(-7, 11), M(2, 1)$ 29. $R(4, -6), M(-7, 8)$ 30. $R(-4, -6), M(3, -4)$

EXAMPLE 4

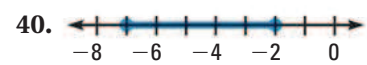
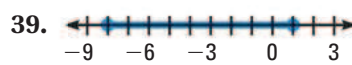
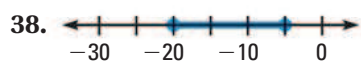
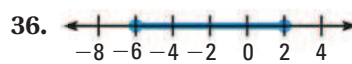
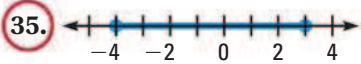
on p. 18
for Exs. 31–34

DISTANCE FORMULA Find the length of the segment. Round to the nearest tenth of a unit.



34. **★ MULTIPLE CHOICE** The endpoints of \overline{MN} are $M(-3, -9)$ and $N(4, 8)$. What is the approximate length of \overline{MN} ?
 (A) 1.4 units (B) 7.2 units (C) 13 units (D) 18.4 units

NUMBER LINE Find the length of the segment. Then find the coordinate of the midpoint of the segment.



41. **★ MULTIPLE CHOICE** The endpoints of \overline{LF} are $L(-2, 2)$ and $F(3, 1)$. The endpoints of \overline{JR} are $J(1, -1)$ and $R(2, -3)$. What is the approximate difference in the lengths of the two segments?
 (A) 2.24 (B) 2.86 (C) 5.10 (D) 7.96
42. **★ SHORT RESPONSE** One endpoint of \overline{PQ} is $P(-2, 4)$. The midpoint of \overline{PQ} is $M(1, 0)$. Explain how to find PQ .

COMPARING LENGTHS The endpoints of two segments are given. Find each segment length. Tell whether the segments are congruent.

43. \overline{AB} : $A(0, 2), B(-3, 8)$ 44. \overline{EF} : $E(1, 4), F(5, 1)$ 45. \overline{JK} : $J(-4, 0), K(4, 8)$
 \overline{CD} : $C(-2, 2), D(0, -4)$ \overline{GH} : $G(-3, 1), H(1, 6)$ \overline{LM} : $L(-4, 2), M(3, -7)$


46. **xy ALGEBRA** Points $S, T,$ and P lie on a number line. Their coordinates are 0, 1, and x , respectively. Given $SP = PT$, what is the value of x ?

47. **CHALLENGE** M is the midpoint of \overline{JK} , $JM = \frac{x}{8}$, and $JK = \frac{3x}{4} - 6$. Find MK .

PROBLEM SOLVING


EXAMPLE 1 □
on p. 15
for Ex. 48

48. **WINDMILL** In the photograph of a windmill, \overline{ST} bisects \overline{QR} at point M . The length of \overline{QM} is $18\frac{1}{2}$ feet. Find \overline{QR} and \overline{MR} .

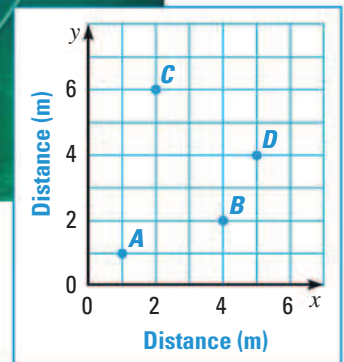
 for problem solving help at classzone.com



49. **DISTANCES** A house and a school are 5.7 kilometers apart on the same straight road. The library is on the same road, halfway between the house and the school. Draw a sketch to represent this situation. Mark the locations of the house, school, and library. How far is the library from the house?

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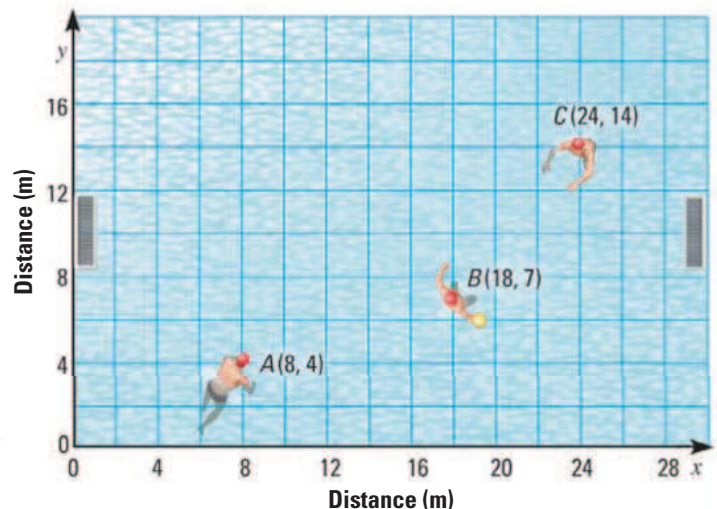
ARCHAEOLOGY The points on the diagram show the positions of objects at an underwater archaeological site. Use the diagram for Exercises 50 and 51.



50. Find the distance between each pair of objects. Round to the nearest tenth of a meter if necessary.
- A and B
 - B and C
 - C and D
 - A and D
 - B and D
 - A and C
51. Which two objects are closest to each other? Which two are farthest apart?

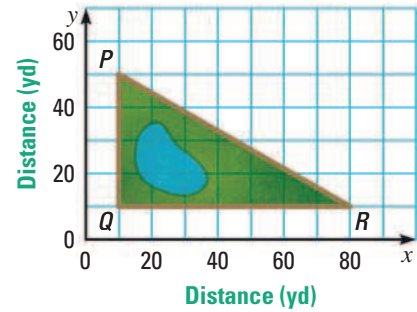
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52. **WATER POLO** The diagram shows the positions of three players during part of a water polo match. Player A throws the ball to Player B, who then throws it to Player C. How far did Player A throw the ball? How far did Player B throw the ball? How far would Player A have thrown the ball if he had thrown it directly to Player C? Round all answers to the nearest tenth of a meter.



53. ★ **EXTENDED RESPONSE** As shown, a path goes around a triangular park.

- Find the distance around the park to the nearest yard.
- A new path and a bridge are constructed from point Q to the midpoint M of \overline{PR} . Find QM to the nearest yard.
- A man jogs from P to Q to M to R to Q and back to P at an average speed of 150 yards per minute. About how many minutes does it take? *Explain.*

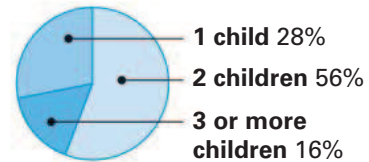


- 54. **CHALLENGE** \overline{AB} bisects \overline{CD} at point M , \overline{CD} bisects \overline{AB} at point M , and $AB = 4 \cdot CM$. Describe the relationship between AM and CD .

MIXED REVIEW

The graph shows data about the number of children in the families of students in a math class. (p. 888)

- What percent of the students in the class belong to families with two or more children?
- If there are 25 students in the class, how many students belong to families with two children?



Solve the equation. (p. 875)

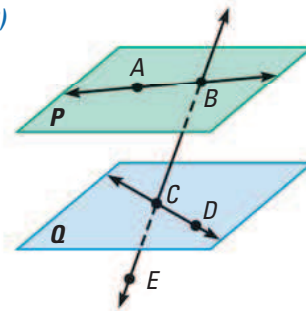
57. $3x + 12 + x = 20$

58. $9x + 2x + 6 - x = 10$

59. $5x - 22 - 7x + 2 = 40$

In Exercises 60–64, use the diagram at the right. (p. 2)

- Name all rays with endpoint B .
- Name all the rays that contain point C .
- Name a pair of opposite rays.
- Name the intersection of \overleftrightarrow{AB} and \overleftrightarrow{BC} .
- Name the intersection of \overleftrightarrow{BC} and plane P .



PREVIEW

Prepare for Lesson 1.4 in Exs. 57–59.

QUIZ for Lessons 1.1–1.3

- Sketch two lines that intersect the same plane at two different points. The lines intersect each other at a point not in the plane. (p. 2)

In the diagram of collinear points, $AE = 26$, $AD = 15$, and $AB = BC = CD$. Find the indicated length. (p. 9)

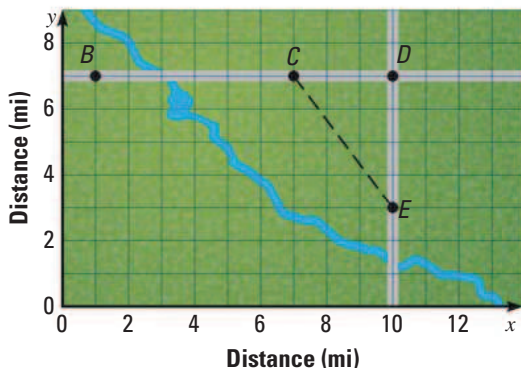


- DE
- AB
- AC
- BD
- CE
- BE
- The endpoints of \overline{RS} are $R(-2, -1)$ and $S(2, 3)$. Find the coordinates of the midpoint of \overline{RS} . Then find the distance between R and S . (p. 15)



Lessons 1.1–1.3

1. **MULTI-STEP PROBLEM** The diagram shows existing roads (\overline{BD} and \overline{DE}) and a new road (\overline{CE}) under construction.



- If you drive from point B to point E on existing roads, how far do you travel?
- If you use the new road as you drive from B to E , about how far do you travel? Round to the nearest tenth of a mile if necessary.
- About how much shorter is the trip from B to E if you use the new road?

2. **GRIDDED ANSWER** Point M is the midpoint of \overline{PQ} . If $PM = 23x + 5$ and $MQ = 25x - 4$, find the length of \overline{PQ} .

3. **GRIDDED ANSWER** You are hiking on a trail that lies along a straight railroad track. The total length of the trail is 5.4 kilometers. You have been hiking for 45 minutes at an average speed of 2.4 kilometers per hour. How much farther (in kilometers) do you need to hike to reach the end of the trail?

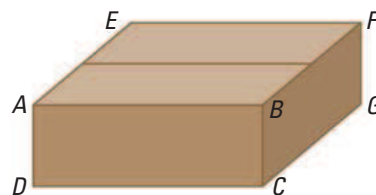
4. **SHORT RESPONSE** The diagram below shows the frame for a wall. \overline{FH} represents a vertical board, and \overline{EG} represents a brace. If $FG = 143$ cm, does the brace bisect \overline{FH} ? If not, how long should \overline{FG} be so that the brace does bisect \overline{FH} ? *Explain.*



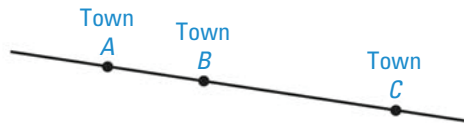
5. **SHORT RESPONSE** Point E is the midpoint of \overline{AB} and the midpoint of \overline{CD} . The endpoints of \overline{AB} are $A(-4, 5)$ and $B(6, -5)$. The coordinates of point C are $(2, 8)$. Find the coordinates of point D . *Explain* how you got your answer.

6. **OPEN-ENDED** The distance around a figure is its *perimeter*. Choose four points in a coordinate plane that can be connected to form a rectangle with a perimeter of 16 units. Then choose four other points and draw a different rectangle that has a perimeter of 16 units. Show how you determined that each rectangle has a perimeter of 16 units.

7. **SHORT RESPONSE** Use the diagram of a box. What are all the names that can be used to describe the plane that contains points B , F , and C ? Name the intersection of planes ABC and BFE . *Explain.*



8. **EXTENDED RESPONSE** Jill is a salesperson who needs to visit towns A , B , and C . On the map below, $AB = 18.7$ km and $BC = 2AB$. Assume Jill travels along the road shown.



- Find the distance Jill travels if she starts at Town A , visits Towns B and C , and then returns to Town A .
- About how much time does Jill spend driving if her average driving speed is 70 kilometers per hour?
- Jill needs to spend 2.5 hours in each town. Can she visit all three towns and return to Town A in an 8 hour workday? *Explain.*